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Nonlinear surface waves on the boundary of a cylindrical electronic medium in the presence of an electric field

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Abstract. It is demonstrated how nonlinear surface modes at the boundary of a cylindrical electronic medium are coupled to external constant or time-varying electric fields in the bounding dielectric. Exact equations governing the interaction are obtained and investigated numerically. It is found that the external field can drive only certain surface modes, in spite of the fact that all the modes are strongly coupled.

1. Introduction

Recently, there has been much interest in the behaviour of surface waves occurring on the boundary of low-temperature plasmas [1–12]. These modes can be of importance in many scientific and technological applications, such as in laboratory plasma production and diagnostics, new sources of light, coherent radiation, and particle beams, solid-state and optical control devices, as well as in machines for plasma-assisted material processing [2, 4, 6, 8, 9]. They serve as the coupler of energy and information between the plasma (active medium) and the outside (bounding medium) environment. It is thus of relevance to investigate in detail the surface modes and their interaction with the volume modes in the plasma.

The possible existence of exact solutions for finite amplitude surface waves propagating on the boundary between a cold plasma and its dielectric container has been pointed out recently [10, 11]. These solutions are exact in the sense that, starting from the conservation equations for the electrons and Maxwell's equations, no approximations of any kind, such as perturbations and *ad hoc* truncations, need to be made in obtaining the *exact* eigenfunctions describing the spacetime behaviour of the wave motion. Such solutions are of special interest because, besides characterizing accurately the nonlinear surface wave physics, they are also useful in verifying various approximation or numerical schemes in the study of wave interactions and instabilities.

In this paper, we consider the effect of an external electric field on nonlinear surface waves [10, 11]. It is shown that steady and non-steady external fields in the bounding dielectric can selectively couple to, modulate, and resonantly amplify certain surface modes, while leaving the other modes unaffected. The coupling has the property that the unaffected modes do not grow, although they are nonlinearly coupled to the amplified modes. The reason is that the equations for the unaffected modes do not contain (the growing) terms associated with the external field or the affected mode. These results can be of relevance to research on novel plasma-driven diodes and transistors as well as the production and control of surface-wave generated low-temperature plasmas.

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2. Electron dynamics

Following [10, 11], let us consider a cylindrical electronic plasma medium in the region $0 < r < R$ with a positively charged immobile background of lattice or heavy ions. The plasma is bounded at $r = R$ by a rigid dielectric of constant permittivity ϵ_d. A spatially constant external electric field $E_0(t)\hat{x}$ is applied to the dielectric. Here, $\hat{x}$ is a unit vector perpendicular to the cylinder axis $\hat{z}$, that is, $x = r\cos\theta$. The evolution of the electron density n is governed by the equation of number density conservation

$$\partial_t n + \nabla \cdot (n\boldsymbol{v}) = 0. \tag{1}$$

Here, the electron bulk velocity $\boldsymbol{v}$ satisfies the cold-fluid momentum conservation equation

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = \frac{q}{m}\boldsymbol{E} \tag{2}$$

where $q = -e$, and m are the electron charge and mass, and $\boldsymbol{E}$ is the wave electric field.

Our formulation is in the same spirit as that of Lorenz [12] who investigated nonlinear waves and found deterministic chaos in atmospheric physics by first separating the spatial variations from the temporal one. However, in contrast to the approach of Lorenz, here no *ad hoc* truncation of any kind needs to be made. Accordingly, for the spatial wave structure inside the nonlinear medium, we make the ansätze

$$n = n(t) \tag{3a}$$

$$v_r = (v_2 + v_1\cos 2\theta)\frac{r}{R} + v_c\cos\theta \tag{3b}$$

$$v_\theta = -v_1\frac{r}{R}\sin 2\theta - v_c\sin\theta + v_3\frac{r}{R} \tag{3c}$$

$$\varphi_{r<R} = (\varphi_c + \varphi_m\cos 2\theta)\frac{r^2}{R^2} - \varphi_c + \varphi_n\frac{r}{R}\cos\theta + \varphi_3\frac{r}{R}\sin\theta \tag{3d}$$

where v_1, v_2, v_3, v_c, φ_3, φ_c, φ_m and φ_n are functions of time only. The above ansätze have been chosen such that they lead to exact equations governing the time evolution of the latter quantities. Thus, one may consider the present approach as a generalized separation of variables method.

For the external and self-consistent fields in the linear bounding dielectric, we have

$$\varphi_{r>R} = -E_0(t)r\cos\theta + (\varphi_n + RE_0)\frac{R}{r}\cos\theta + \varphi_m\frac{R^2}{r^2}\cos 2\theta - \varphi_3\frac{R}{r}\sin\theta \tag{4}$$

which satisfies the Laplace equation as required. The external electric field $E_0(t)$ may be constant or any function of time. Thus, the *far-field* electric field in the bounding dielectric has the structure

$$\boldsymbol{E}_{r\gg R} = -\nabla\varphi_{r\gg R} = E_0(t)\hat{x} + \mathrm{O}\left(1/r\right)\hat{\theta} + \mathrm{O}\left(1/r^2\right)\hat{r} \tag{5}$$

where we emphasize that $\hat{x}$ is a unit vector perpendicular to the axis of the cylinder.

The equations are completed by Poisson's equation

$$\nabla^2\varphi = -\frac{q}{\epsilon_0}(n - n_0) \tag{6}$$

where n_0 is the constant background ion density.

We shall also need the boundary condition that the total current density is continuous across the interface between the plasma and the rigid dielectric, namely

$$\left[qnv_r - \epsilon_0 \partial_t \partial_r \varphi\right]_{r=R-0} = \left[-\epsilon_0 \epsilon_{\rm d} \partial_t \partial_r \varphi\right]_{r=R+0}.$$

Consistent with the cold-plasma approximation, the thickness of the surface layer at the interface is taken to be smaller than any other characteristic dimension. We note that for a warm or hot plasma, a surface layer with a thickness of the order of the Debye length can appear near $r = R - 0$. In such a layer highly localized surface charges and currents can occur.

3. Evolution equation

The ansätze (3) have been chosen such that the spatial and time dependences of the original field variables can be separated. In fact, substituting (3) into (1), (2) and (6), using the boundary condition and equating the coefficients of the various spatial variables, one obtains

$$d_t N + 2NV_2 = 0 \tag{7}$$

$$d_t V_1 + 2V_1 V_2 = -2\phi_m \tag{8}$$

$$d_t V_2 + V_1^2 + V_2^2 - V_3^2 = \tfrac{1}{2}(N-1) \tag{9}$$

$$d_t V_3 + 2V_2 V_3 = 0 \tag{10}$$

$$d_t V_c + (V_1 + V_2)V_c = -\phi_n \tag{11}$$

$$4\phi_c = 1 - N \tag{12}$$

$$2(1+\epsilon_{\rm d})d_t\phi_m = NV_1 \tag{13}$$

$$(1+\epsilon_{\rm d})d_t\phi_n = NV_c + 2\epsilon_{\rm d} d_t \tilde{E}_0 \tag{14}$$

where we have defined $N = n/n_0$, $V_j = v_j/R\omega_{\rm p}$ $(j = 1, 2, 3, c, m, n)$, $\phi_j = \epsilon_0\varphi_j/n_0 q R^2$, $\tilde{E}_0 = \epsilon_0 E_0/n_0 R q$, and the time t has been normalized by the inverse plasma frequency $\omega_{\rm p}^{-1}$.

Since ϕ_c is given directly by (12) in terms of N, and from (7) and (10) $V_3 = CN$, where C is a constant, we have effectively six unknowns, N, V_1, V_2, V_c, ϕ_m, and ϕ_n governed by six nonlinear ordinary differential equations. We stress that no approximations of any kind have been made in obtaining this set of nonlinear evolution equations from the basic equations (1), (2) and (6), which are fairly general, together with the boundary conditions. The solutions, on the other hand, are particular because of the ansätze (3) used in order to achieve the separation of the space and time variables.

4. Nonlinear waves

Before proceeding with the numerical solutions, it is instructive to briefly discuss the linear limit first. Here, one finds that the variable pairs N and V_2, V_1 and ϕ_m, and V_c and ϕ_n are the eigenfunctions of three independent *linear* modes of the simple harmonic type. The first (N, V_2) is a volume mode with the (normalized) frequency unity, representing the ordinary plasmons. The others, (V_1, ϕ_m) and (V_c, ϕ_n), are surface plasmon modes with the frequency $(1+\epsilon_d)^{-1/2}$. We note that the external field E_0 is coupled directly only to the (V_c, ϕ_n) surface mode.

For an insight into the mode structures under discussion, let us look at the special case $\epsilon_d = 3$. Here, the (normalized) frequencies of the three natural modes involved are 1, 1/2 and 1/2, so that a three-wave resonance would seem to be possible. This actually does not occur. It is easily seen from the equations that the nonlinear terms in (7)–(14) do not satisfy the three-wave coupling conditions, so that they do not couple the appropriate modes for resonant interaction to occur despite the matching of the frequencies of the three modes.

Using the Runge–Kutta method for numerical integration, we have solved the evolution equations for several representative cases. In the integration, we start with small initial values for all the variables. These initial conditions determine the final states of the waves. Since we have not been able to derive any useful conservation laws or analytical existence conditions for the solutions, we proceed to solve the equations using various sets of initial conditions. Solutions which become numerically unstable and/or are unphysical are discarded. Figure 1(a) shows the well behaved volume and surface waves for the case $E_0 = 0$. Figure 1(b) shows the corresponding phase diagrams. For this case as well as those following, we have used $\epsilon_d = 1$ and $C = 0.02$. It is worth noting that, although the amplitudes of the waves are small, frequency shift and broadening due to the nonlinear wave–wave interaction are evident from the band-like structures of the phase diagrams of the three waves, indicating strong nonlinear coupling. In figure 2, E_0 is taken to be sinusoidal in time, namely $\tilde{E}_0 = A\sin(\omega_0 t)$, where $A = 0.05$. Note that both the amplitude and the frequency of the surface mode (V_c, ϕ_n) are modulated in a stable (non-growing) manner. That is, the waves are slightly frequency and amplitude modulated without the occurrence of an instability. Figure 3 shows the case in which the external field is of small amplitude, namely $A = 0.01$, but oscillates at the resonant frequency $(1+\epsilon_d)^{-1/2} \approx 0.71$ of the linear surface waves. One sees that here resonance growth of the affected surface mode occurs, but the other two modes are left unchanged. We have also evaluated the case (not shown) in which the external field is a constant, representing a constant energy input, and obtained the expected result that the affected mode grows linearly without additional frequency modulation. Furthermore, larger amplitude waves with broad frequency bands can also be shown to exist.

We emphasize that in the present formulation the structure of the evolution equations is such that only one mode, namely that corresponding to the surface fluctuations (V_c, ϕ_n), is coupled directly to the external electric field and can become resonant with it. The other modes, which are nonlinearly coupled to this surface mode, can affect it although not vice versa.

5. Discussion

We have demonstrated that exact nonlinear wave solutions can be constructed for a cylindrical electronic medium bounded by a dielectric in an external electric field

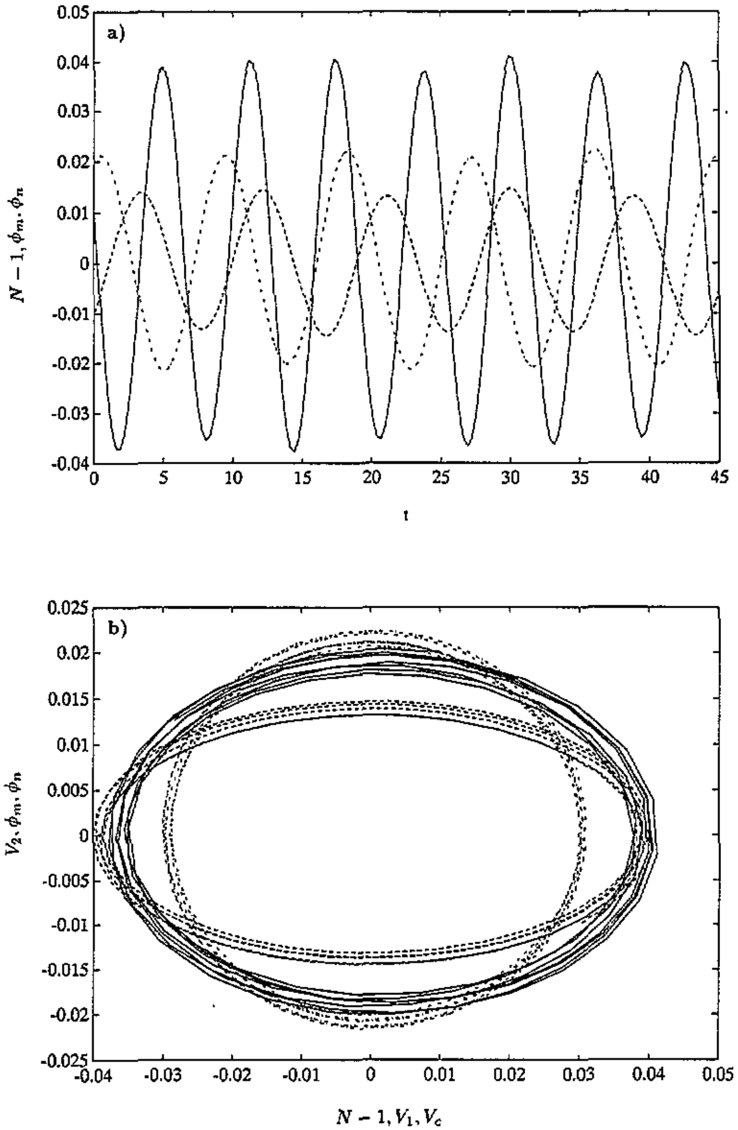

Figure 1. (a) Small amplitude waves in the absence of $\tilde{E}_0$, amplitude against time. Full curve, $N-1$; broken curve, ϕ_m; and chain curve, ϕ_n. (b) Phase space of the three modes. Full curve, $(N-1, V_2)$; broken curve, (V_1, ϕ_m); and chain curve, (V_c, ϕ_n). Note the nonlinear frequency broadening despite the small amplitudes. Here and below, $\varepsilon_d = 1$ and $C = 0.02$.

perpendicular to the axis of the cylindrical plasma. Linearly, three distinct modes, one volume and two surface, exist near the plasma boundary. Nonlinearly, these modes are strongly coupled, although not resonantly. Furthermore, only one of the modes, a surface plasmon mode, can be driven by the external field. This mode, which is directly coupled to the external field, can become unstable if the latter oscillates at a frequency near the resonance frequency $(1+\epsilon_d)^{-1/2}\omega_p$ of the surface plasmon. The other modes can in principle feed energy into the mode which is affected. Nevertheless, they cannot be driven by the external field or the growing mode.

We have assumed that the applied field is maintained by some external sources. If it is

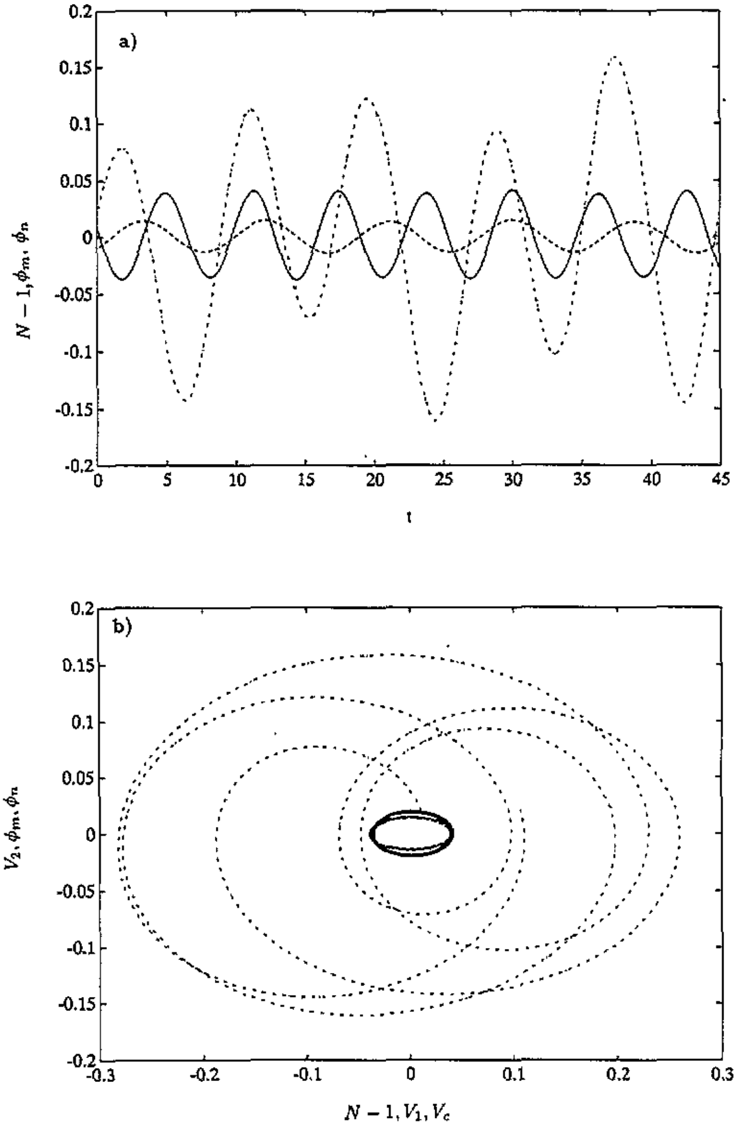

Figure 2. (*a*) Waves in the presence of $\tilde{E}_0 = A\sin(\omega_0 t)$, with $A = 0.05$ and $\omega_0 = 0.3$. The latter is much lower than the normalized resonance frequency, which is approximately 0.71 for $\epsilon_d = 1$. (*b*) Phase space. Note the stable frequency and amplitude modulation of the third mode (chain curves).

allowed to evolve self-consistently according to the properties of the dielectric, we expect its amplitude to decrease as it feeds energy to the growing surface waves, leading eventually to a steady state.

Our results may be of interest to the study of plasma diodes and transistors, surface-wave generated plasmas, plasma–wall interaction, control of plasmas for material processing, modulation of pulses in fibre-optics communications, as well as for the verification of approximation and numerical methods in nonlinear wave problems. Since the evolution equations are obtained without making use of perturbations and truncations of any kind, the system may provide another mathematically exact model for investigating wave instabilities,

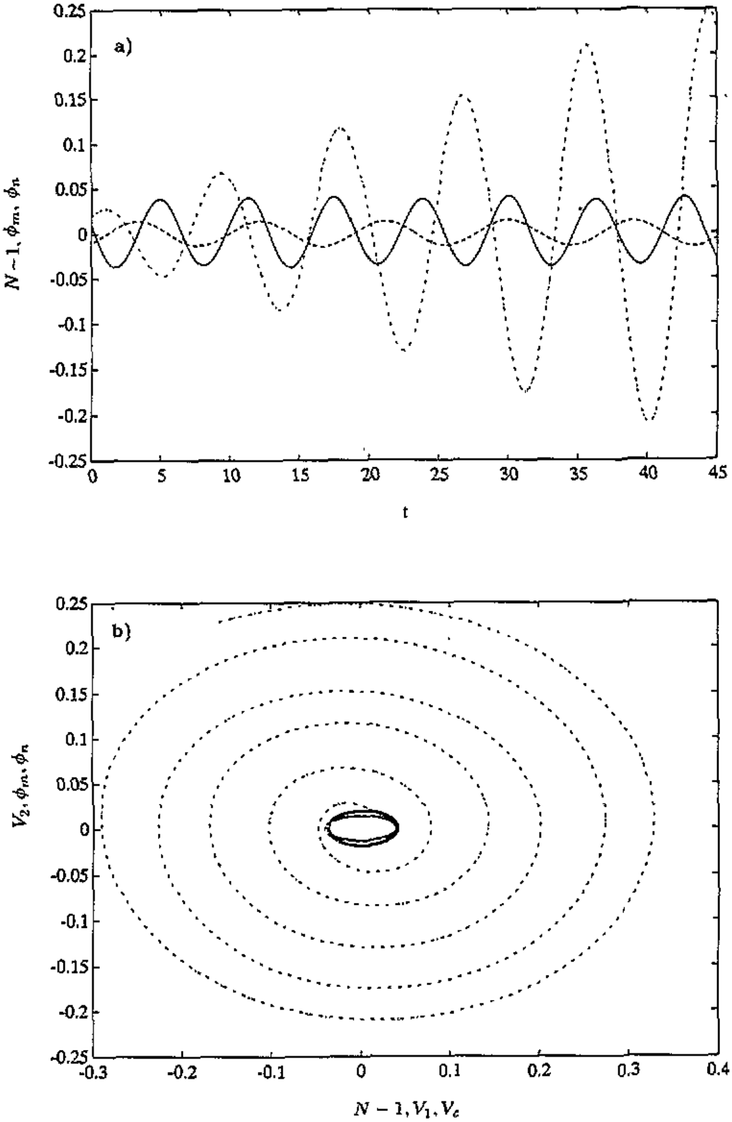

Figure 3. Same as figure 2, except that $A = 0.01$ and $\omega_0 = 0.71$. Here, although A is smaller, fast growing unstable resonance of the third mode (chain curves) occurs.

saturation, and deterministic chaos [12]. On the other hand, at present we are unaware of any more general ansätze for the field quantities which can lead to more general nonlinear wave problems.

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